A functional approximation for the phase-type distributions single-server queues

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Abstract. The use of queuing models is a basic tool for studying systems involving contention for resources. Major application areas include computing systems, telecommunication systems, and manufacturing systems. Queuing models are important tools for studying the performance of complex systems, but despite the substantial queuing theory literature, it is often necessary to use approximations in the case the system is non-markovian. Phase type distribution is by now indispensable tool in creation of queuing system models [4,5,2,3].

In the majority of the cases, all model parameters are imprecisely known because they are determined through insufficient statistical data (a finite number of observations). leading to uncertainty in the assessment of their values. This parametric uncertainty induced from the incomplete information concerning the parameter is called "epistemic uncertainty" [6].

this paper proposes a numerical approach based on Taylor series expansion [1,7,8] with a statistical aspect for analyzing the stationary performances of the phase-type distributions single-server queues and finite orbit ($M/PH/1/N$), where we assume that the parameter $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ of the service time is not assessed in a perfect manner. Additionally, approximate expressions of the probability density functions, the expectation and the variance of the performance measures are obtained and compared to the corresponding Monte Carlo simulations results.

Keywords: Queueing systems, Taylor-series expansions,phase-type distributions, Perturbation, Sensitivity analysis, performances measures..

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