On stability of two-class retrial queue with constant retrial rates and general service times

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The paper deals with a two-class retrial system with Poisson inputs, class-dependent constant retrial rate and general state-dependent service times. In such a model a customer, who meets busy server, is blocked on a class-dependent orbit and then tries to occupy server after (class-dependent) exponential retrial times. To obtain the stability criterion for the system under consideration, we apply the technique developed in the book Fayolle et al. [1] for analysis of the multidimensional Markov chains. The research also uses the regenerative approach well presented at instance in Asmussen [2], Morozov [3]. Note that stability conditions of this system with exponential service times have been obtained by other methods in the paper Avrachenkov et al. [4].

In the model under consideration arrivals form the superposition of two Poisson inputs with corresponding rates λ_i, where $i = 1, 2$ defines the class number. Next denote the corresponding orbit rates by α_i. Moreover we assume that class-i service times are iid generally distributed and stochastically equivalent to $S^{(i)}$. Thus we have the marginal load coefficients $\rho_i = \lambda_i ES^{(i)}$, $\hat{\rho}_i = \alpha_i ES^{(i)}$ and the total load coefficient as follows: $\rho = \rho_1 + \rho_2$.

A key element of the research is an observation that the two-dimensional process, associated with orbit sizes at departure instants, forms an embedded discrete-time Markov chain. This allows us to obtain stability conditions in the terms of the drift of each component of the basic two-dimensional process. From this point of view the stability is equivalent to the ergodicity of the embedded Markov chain. Namely we obtain that two-class retrial system with constant retrial rates, Poisson inputs, general service times and exponential retrials is ergodic, if and only if

$$\rho < \min\Big(\frac{\hat{\rho}_1}{\rho_1 + \hat{\rho}_1}, \frac{\hat{\rho}_2}{\rho_2 + \hat{\rho}_2}\Big).$$

Moreover we explore the phenomenon of partial stability, first detected in the paper Avrachenkov et al. [5], where K-class retrial system has been studied with arbitrary number of classes $K \geq 1$ and a special class of service time distributions. We obtain the conditions, which guarantee that one orbit stays tight while the other increases unlimitedly in probability.

Acknowledgements The study was carried out under state order to the Karelian Research Centre of the Russian Academy of Sciences (Institute of Applied Mathematical Research KRC RAS).

The research is partly supported by Russian Foundation for Basic Research, projects 19-07-00303, 19-57-45022.

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