On Reliability of a Double Redundant Renewable System with Arbitrarily Distributed its Units Life and Repair Times

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In the papers [1, 2] the reliability of a title system with discipline of partial repair after its failure has been considered. Recently the same model with the full system repair after its full failure has been studied [3]. The results of these investigations has been obtained with the help of the theory of Decomposable Semi-Regenerative Processes (DSRP), which has a long history, and its review can be found in [4]. The results was represented in terms of modified Laplace-Stiltjes Transforms (MLST's) cumulative distribution functions (c.d.f.'s) of the system units life- and repair times. In the talk both results will be done and the methods of their obtaining will be demonstrated.

1. The model

Consider a homogeneous title system. The system fails when both of units are in a failed state.

For the renewable system, at least two different disciplines of its repair are possible: the partial and the full repair discipline. Denote by $A_i\, B_i$ $(i = 1, 2, \dots)$ the life- and repair times of the system units, and suppose that all these random variables (r.v.'s) are mutually independent and identically distributed. Thus, denote by $A(t) = \mathbb{P}\{A_i \le t\}$ and $B(t) = \mathbb{P}\{B_i \le t\}$ the corresponding c.d.f.'s. Suppose that the instantaneous failures and repairs are impossible and their mean times are finite:

$$A(0) = B(0) = 0, \quad a = \int_0^\infty (1 - A(x))dx < \infty, \quad b = \int_0^\infty (1 - B(x))dx < \infty.$$

Denote by $E = \{i = 0, 1, 2\}$ the set of system states, where i stands for the number of failed units, and introduce a random process $J = \{J(t),\, t \ge 0\}$, where

$$J(t) = \{\text{number of failed units at time } t.\}$$

In the talk the reliability function $R(t)$, time-dependent system state probabilities (t.d.s.p.'s) $\pi_j(t) = \mathbb{P}\{J(t) = j]\}$ $(j = 0, 1, 2)$, the steady state probabilities (s.s.p.'s) $\pi_i = \lim\limits_{t\to\infty} \pi_j(t)$ $(j = 0, 1, 2)$, are considered

2. Modified Laplace-Stiltjes Transform

The following notations will be used next.

- The Laplace–Stiltjes transforms (LSTs) of the c.d.f.'s are denoted as:

$$\tilde{a}(s) = \mathbb{E}\left[e^{-sA}\right] = \int\limits_0^\infty e^{-sx} dA(x), \quad \tilde{b}(s) = \mathbb{E}\left[e^{-sB}\right] = \int\limits_0^\infty e^{-sx} dB(x),$$

- The modified Laplace-Stiltjes Transforms (MLST's) are denoted as

$$\tilde{a}_B(s) = \int_0^\infty e^{-sx} B(x) dA(x), \qquad \tilde{b}_A(s) = \int_0^\infty e^{-sx} A(x) dB(x). \quad (1)$$

- Corresponding truncated expectations are denoted by:

$$a_B = \int_0^\infty x B(x) dA(x), \quad b_A = \int_0^\infty x A(x) dB(x). \quad (2)$$

- The probabilities $\mathbb{P}\{B \leq A\}$ and $\mathbb{P}\{B \geq A\}$ are associated with MLST's as:

$$\tilde{a}_B(0) = \mathbb{P}\{B \leq A\} \equiv p, \quad \tilde{b}_A(0) = \mathbb{P}\{B > A\} \equiv q = 1 - p.$$

- Note the property of transformations (1)

$$\tilde{a}_{1-B}(s) = \tilde{a}(s) - \tilde{a}_B(s), \quad \tilde{b}_{1-A}(s) = \tilde{b}(s) - \tilde{b}_A(s). \quad (3)$$

3. Main results for the model with partial repair discipline

The main results for the system study with partial repair discipline contain in the following theorems.

Theorem 1. *The LT $\tilde{R}(s)$ of the system reliability function $R(t)$ is*

$$\tilde{R}(s) = \frac{(1 - \tilde{a}(s))(1 + \tilde{a}(s) - \tilde{a}_B(s))}{s(1 - \tilde{a}_B(s))}. \quad (4)$$

From the theorem it follows that the mean system lifetime is $\tilde{R}(0) = \frac{a}{q}$.
For LTs of the t.d.s.p.'s has been obtained in [?], however, they have rather complex expressions and will be presented in the talk. The appropriate s.s.p.'s has been found in [?] and given in the following theorem

Theorem 2. *The system state stationary probabilities are:*

$$\pi_0 = \quad 1 - \frac{b}{a_B + b_A}, \quad \pi_1 = \frac{a+b}{a_B + b_A} - 1, \quad \pi_2 = 1 - \frac{a}{a_B + b_A}. \quad (5)$$

4. Main results for the model with full repair discipline

The main results for the system with full repair discipline has been found in [3] and the expression for the LST $\tilde{R}(s)$ of the system reliability function $R(t)$ coincide with (4) for the system with partial repair discipline.

However the expressions for the t.d.s.p.'s and the s.s.p.'s differs from those for the system with partial repair discipline. We omit here the expressions for t.d.s.p.'s (they will be represented in the talk and the full paper), and represent only expressions for the s.s.p.'s

Theorem 3. *The s.s.p.'s for the considered system with full repair discipline are:*

$$\pi_0 = \frac{aq + a_B + b_A - b}{a + q(a+c)}, \quad \pi_1 = \frac{a + b - (a_B + b_A)}{a + q(a+c)}, \quad \pi_2 = \frac{cq}{a + q(a+c)}. \tag{6}$$

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