Accumulative pension schemes with limited inheritance period and various decrement factors

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In this work, the authors continue their earlier investigation on accumulative pension schemes [1–3]. In particular, the authors generalize the results of [1] for the case of accumulative pension schemes with a limited inheritance period.

Accumulative pension schemes are popular in many countries (see [4, 5]). Below we propose two pension schemes with one decrement factor (mortality) and two decrement factors (mortality and disability). For each scheme, we construct the balance equation and obtain an expression for calculation of gross premium. The obtained results are of practical importance for the Pension Fund of the Russian Federation, as well as for private pension funds and pension actuaries in general.

1. Accumulative pension scheme with one decrement factor

Consider an accumulative defined contribution pension scheme with a lump sum payment on retirement. It is assumed that contributions are paid regularly at the beginning of each year.

Let us introduce the following notation and assumptions (for more details see [1–3])

x is the age of the insured person at the time of concluding the pension insurance contract.

y is the retirement age.

B is the gross premium paid by the policyholder at the beginning of year until retirement.

P_L is the lump sum payment on retirement if the insured person survives to the retirement age.

$\alpha_1, \ldots, \alpha_{y-x}$ are the annual premium loads under the insurance pension contract.

i is the effective annual interest rate and $\nu = \frac{1}{1+i}$ is the annual discount factor.

P_r is the present value of the net contributions (i.e., the contributions balance after taking into account the premium loads and income from investments) refunded to the inheritors if the insured dies before surviving to the retirement age and the death occurs within a limited inheritance period $(x + l, y)$. It is assumed that the contributions are not refunded if the death occurs within $(x, x + l)$.

The balance equation for this scheme has the form

$$B\sum_{k=0}^{y-x-1}\nu^{k}{}_{k}p_{x}=B\sum_{k=0}^{y-x-1}\alpha_{k+1}\nu^{k}{}_{k}p_{x}+\nu^{y-x}{}_{y-x}p_{x}P_{L}+P'_{r}, \tag{1}$$

where (see [6, 7] for more details about the actuarial symbols)

$$P'_{r}=\frac{P_{r}}{l_{x}}=B\sum_{j=l}^{y-x-1}{}_{j|}q_{x}\sum_{k=1}^{j+1}(1-\alpha_{k})\nu^{k-1}, \tag{2}$$

l_z is the expected number of active insured persons of the pension scheme at age z,

${}_{t}p_{z}=l_{z+t}/l_{z}$ is the probability that (z) survives at least to time t (and dies some time after t). Here (z) refers to an individual alive at age z,

${}_{t|}q_{z}=\frac{l_{z+t}-l_{z+t+1}}{l_{z}}$ is the probability that (z) will die in a year, deferred t years; that is, that he will die in the $(t+1)$th year

In particular, the annual gross premium B is equal to

$$B=\frac{\nu^{y-x}{}_{y-x}p_{x}P_{L}}{\sum\limits_{k=0}^{y-x-1}\nu^{k}{}_{k}p_{x}-\sum\limits_{k=0}^{y-x-1}\alpha_{k+1}\nu^{k}{}_{k}p_{x}-\sum\limits_{j=l}^{y-x-1}{}_{j|}q_{x}\sum\limits_{k=1}^{j+1}(1-\alpha_{k})\nu^{k-1}}. \tag{3}$$

2. Accumulative pension scheme with two decrement factors

For this scheme, the conditions of the insurance contract in the event of death of the insured are the same as in the first scheme. If the disability occurs before the age $y-1$ then the net contributions are refunded to the insured. If the insured person has become disabled within $(y-1,y)$ and reaches the retirement age y, then the pension fund pays him the lump sum at age y.

The balance equation for this scheme has the form

$$B\sum_{k=0}^{y-x-1}\nu^{k}{}_{k}p_{x}=B\sum_{k=0}^{y-x-1}\alpha_{k+1}\nu^{k}{}_{k}p_{x}+\nu^{y-x}{}_{y-x}p_{x}P_{L}+P'_{r}, \tag{4}$$

where

$$P'_{r}=\frac{P_{r}}{l_{x}}=B\left[\sum_{j=0}^{l-1}{}_{j|}q'_{x}\sum_{k=1}^{j}(1-\alpha_{k})\nu^{k-1}+\sum_{j=l}^{y-x-1}{}_{j|}q''_{x}\sum_{k=1}^{j+1}(1-\alpha_{k})\nu^{k-1}\right], \tag{5}$$

where

${}_{s}p_{x}$ is the probability for an active insured aged x, of being active at age $x+s$

${}_{s|}q'_{x}$ is the probability for an active insured aged x, of being active at age $x+s$ and leaving the pension scheme for reason of disability during the next year (i.e., between ages $x+s$ and $x+s+1$)

${}_{s|}q''_x$ for $s = 1, \ldots, y - x - 2$ is the probability for an active insured aged x, of being active at age $x + s$ and leaving the pension scheme for reason of death or disability during the next year (i.e., between ages $x + s$ and $x + s + 1$)

${}_{y-x-1|}q''_x$ is the probability for an active insured aged x of being active at age $y - 1$ and leaving the pension scheme for reason of death during the next year.

References

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