Extreme weights for constant proportion index portfolio strategies

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It is well known that two main types of strategies are used in the practice of investing: active and passive. Active strategies involve continuous analysis of the market, forecasting the prices (yields) of assets and restructuring the portfolio (buy-sell) of assets. Such strategies are very expensive (costs for commissions, fees for processing and analyzing information and, in fact, costs for management).

Passive (index) strategies consist in copying the index [1]. Such strategies are systematically implemented by the so-called index funds that accumulate the funds of investors (Depositors Fund) and invest them in a widely diversified market portfolio of assets.

Index funds have recently gained more acceptance among both private and institutional investors. The impetus for the emergence and spread of such strategies was the creation of the so-called CAPM model (Capital Asset Price Model) in the 60-70 years of the last century by a group of American economists (Tobin, Sharp, Mossin and oth.). Within the framework of CAPM Tobin proves the famous two-fund theorem: any effective and, therefore, return or risk optimal portfolio π is a combination of a risk-free asset with a weight w_0 and a market portfolio M with a weight w_M. The return of such portfolio is equal to ([2-5])

$$r_\pi = w_0 r_0 + w_M r_M \tag{1}$$

and risk (standard deviation)

$$\sigma_\pi = w_M \sigma_M, \tag{2}$$

where $w_0 + w_m = 1$.

Tobin's theorem played an important role in the investment business. It became clear that if one creates a well-diversified Fund that allocates the capital in a wide range of assets then the problem of the optimal portfolio construction is reduced simply to choose the optimal proportion between risk-free and risky (market) parts of π, i.e. to determine the two weights w_0 and w_M. In practice, index funds have become such funds.

In common constant proportion strategies the investor simply chooses some level of investment in risky assets. By choosing the proportion of invested capital in the market portfolio (in practice the market portfolio is some kind of index fund), the investor maintains this level constant for a sufficiently long investment period. Since the value of the index fund share is changing over

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time the proportion of the invested capital in the index fund is also changing. Hence it is necessary to rebalance the portfolio for maintaining the constant weight of index fund in the portfolio.

The aim of this work is to evaluate and analyze the optimal index portfolio strategies for Russian index funds. Five index funds with largest assets net value are selected. Namely we select funds operating in the market for at least 10 years and "well-tracking" MICEX index (with the coefficient of determination R2 and beta close to 1): RGS, Otkrytie, VTB, Raiffeisen, and one large fund Alpha Capital with a fairly low coefficient of determination and beta.

According to our results for all considered funds and all investment periods (ranged from one to ten years and with monthly, quarterly or annual rebalancing), the optimal weights of the index portfolios in almost all cases take the extreme values 0 or 1. Intermediate values of optimal weights are extremely rare.

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