Steady state performance comparison of second and third order filters

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The problem design unbiased functional filters (optimal functional observers) of the second and third orders is considered. The scalar linear functional of the state vector of the control plant with stochastic disturbances is estimated from the measured scalar output. Perturbations are white random processes uncorrelated with each other at different times and with the initial state of the system. As a criterion of optimality and comparison of the constructed filters of the second and third orders, the root-mean-square error in the steady-state mode is chosen. To calculate the criterion, the method of integral quadratic quality assessments is used. The scalar linear functional of the state vector of a system with stochastic disturbances is chosen in such a way that there is no optimal first-order filter.

In practice, optimal filters are used in the signal processing of satellite navigation systems, in process management, in economics, in the prediction of earthquakes and cyclones, in the analysis of water resources, and elsewhere.

The problem under study is at the intersection of two classical problems of automatic control theory: the problem of optimal observation of full order and the problem of constructing a functional observer for deterministic systems. The first problem relates to the theory of optimal filtration and was first solved in 1960-61 by Kalman and Bucy [1, 2] for both continuous and discrete time. The solution to the second problem of constructing functional observers for linear stationary fully defined systems was first proposed in 1966 by Luenberger [3]. The further development of the theory of functional observers is reflected in detail in the books by O'Reilly [4], Korovin and Fomichev [5]. In particular, in [5] the conditions of existence and algorithms for the synthesis of functional observers for linear stationary fully deterministic systems are given for various cases, namely, scalar and vector inference, scalar and vector functional. There are also two methods for solving the problem: the pseudo-input method and the scalar observer method. Both methods allow one to obtain necessary and sufficient conditions for the existence of functional observers of order k ($k < \nu - 1$, where ν is the observability index of the system) which were first proposed in [6, 7].

Recently, much attention has been paid to the construction of low-order filters for linear systems. In [8–10], necessary and sufficient conditions for the existence and uniqueness of functional optimal observers for linear continuous (in [10] for both continuous and discrete) systems with noise, and methods for constructing such observers in the time domain are given. The method of constructing a reduced-order filter proposed in the paper [9], is based on the transformation of the original stochastic system. The results obtained in [10] generalize the results obtained in [8, 9]. In [11], a method for constructing unbiased low-order filters for stochastic discrete (both stationary and non-stationary) systems with unknown inputs is presented. The necessary and sufficient conditions for the existence of the resulting filter are given. Stability conditions are obtained for stationary systems. In [12], a method for the synthesis of functional optimal observers in the frequency domain using spectral factorization in continuous and discrete time is proposed, the transfer function of the filter and the properties of its associated innovation sequence are obtained. In [13], a generalization of the classical unbiasedness condition in the joint problem of stabilization and optimal filtering is presented, and an alternative method for constructing low-order filters is proposed, based on reduction to a nonlinear optimization problem.

In this article, instead of the traditionally used Kalman filter, which forms an estimate of the total system state vector and has an order that coincides with the order of the system, it is proposed to construct its analogue, a functional filter with a reduced dimension. In this case, both the demand for the resources of the computing device on which the functional filter is implemented and the computation time will decrease. In addition, the low order of the system makes it possible to simplify the analysis of the dynamic system.

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