Limit theorems for random flights

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Random flights represent an interesting class of random walks in a random environment. In general terms, they can be described as follows: it is the moving particle process in R^d which is defined by two independent sequences (T_k) and (θ_k) of random variables. The variables T_k are non negative and for all k $T_k \leq T_{k+1}$, while d-dimensional variables θ_k form an i.i.d sequence. The values $\frac{\theta_k}{|\theta_k|}$ are interpreted as the directions, and T_k as the moments of change of directions.

A particle starts from zero and moves in the direction θ_1 up to the moment T_1 with the constant speed equal to $|\theta_1|$. It then changes direction to θ_2 and moves on within the time interval of length $T_2 - T_1$ with the constant speed equal to $|\theta_2|$, etc. The position of the particle at time t is denoted by $X(t)$.

In recent paper [1] Davydov and Konakov studied the global behavior of the process $X = \{X(t),\ t \in R_+\}$, namely, they where looking for conditions under which the processes

$$Y_T(t) = \frac{1}{B(T)} X(tT), \quad t \in [0,1],$$

weakly converges in $C[0,1]:\ Y_T \Longrightarrow Y,\ B_T \longrightarrow \infty,\ T \longrightarrow \infty.$

It was supposed that the points (T_k), $T_k \leq T_{k+1}$, have the form $T_k = f(V_k)$ where (V_k) is an homogeneous Poisson point process in R_+.

It was shown that in dependence of the growth of f there are three different types of limiting processes:

- If the function f has power growth, the behavior of the process is analogous to the uniform case and then in the limit we obtain a Gaussian process which is a linearly transformed Brownian motion.
- In the case of exponential growth, the limiting process is piecewise linear with an infinite number of units, but $\forall \epsilon > 0$ the number of units in the interval $[\varepsilon, 1]$ will be a.s. finite.
- Finally, with the super exponential growth of f, the process degenerates: its trajectories are a.s. linear functions.

The aim of my talk is to present new results showing that these three types of convergence are preserved when replacing the particular random walk (V_n) with a sequence of consecutive sums of a very large class of strictly stationary process.

References

[1] Yu. Davydov and V. Konakov, Random walks in non homogeneous Poissonian environment. in : Modern problems of stochastic analysis and statistics - Selected contributions in honor of V. Konakov, Vol. 208, Heidelberg : Springer, pp. 3–24.